

# NAFF

## hpNaff

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### Numerical Analysis of Fundamental Frequencies (NAFF)

A little command line tool (macOS, Linux, Windows) to perform high-resolution frequency analysis of time series following the Laskar method.

Pre-compiled binaries are available at <https://paloz.marum.de/bitbucket/projects/ESSP/repos/hpnaff/browse/Binaries>.



#### Note for Windows

Windows version requires installation of [Windows Swift Toolchain](#)

For macOS:

after download, copy 'hpnaff' to a location in your search path,

and add execute permissions by 'chmod a+x ./hpnaff; cp hpnaff /usr/local/bin'.

For convenience: a code-signed .dmg file is also available here: [hpNaff-0.4.0.dmg](#)

Then try it on an [example file](#) that contains a weighted mix of Earth's orbital eccentricity, obliquity and climatic precession.



#### Prototype WebApp version

A prototype (alpha) version (without any of the options) of this tool via a WebApp is available at <https://paloz.marum.de/hpNaffWeb/index.html>

Please drag and drop a tab-separated textfile (two columns only, evenly spaced) onto the area, adjust the time/depth step, and press compute.

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### Usage:

```
% ./hpnaff -v
Welcome to hpNAFF Analysis. ©2015-2021 Heiko Pälke. 3rd party licenses: see --licenses option
Please provide exactly one file name argument
Usage: hpnaff [options] file
file format: tab separated columns, data must be equidistantly spaced. Instead of file can specify '-' for stdin.
-h, --help:
  Displays help message.
-r, --fracRMSChangeLimit:
  fracRMSChangeLimit. Default: 0.000002
-f, --fracFreqAccuracyLimit:
  fracFreqAccuracyLimit. Default: 0.00000001
-m, --maxFrequencies:
  maxFrequencies. Default: 30
-c, --freqCycleLimit:
  freqCycleLimit. Default: 100
-l, --lowerFreqLimit:
  lowerFreqLimit. Default: 0.0
-u, --upperFreqLimit:
  upperFreqLimit. Default: 0.5 (NYQUIST)
-d, --detrendingOrder:
  Order for polynomial detrending. 0 removes DC only. >0 removes a polynomial fit. Default: not set
-dC, --detrendChunks:
  Option to apply detrending (-d) to individual chunks. Default: false
-fAL, --filterAbscissaLower:
  filter on Abscissa (x-axis) data greater or equal than value specified.
-fAU, --filterAbscissaUpper:
  filter on Abscissa (x-axis) data lesser or equal than value specified.
-s, --chunkSize:
  Split the input file into chunks of this size. Default: not set, no chunking
-w, --window_offsets:
  Specify windowing offset for evolutive analysis. Default: chunkSize if set, or None.
-z, --columnSeparator:
  Use this as columns separator for input file instead of default tab \t. Default: \t
-k, --skipHeaderLines:
  Specify number of (header)lines to be skipped. Default: 0
-n, --decimate:
  Specify decimation of input data. Default: 1 (take every value)
-a, --abscissaColumn:
  Specify column number for abscissa (time or depth). Default: 1
-o, --ordinateColumn:
  Specify column number for ordinate (data). Default: 2
-C, --complex:
  Compute spectra for complex input. Default: false. If true, by default takes values from abscissa column +
1.
-Co, --ComplexOrdinateColumn:
  Specify column number for ordinate (data), complex part. Default: 3 (if C option used)
-t, --deltaT:
  Specify time/depth offset between data. Default: 1.0
-x, --ratios:
  Compute and output frequency ratio matrix. Default: false.
--ratio_list:
  If ratio_option is set, provides comma separated list of frequency ratios to highlight in output.
--output_detrended_data:
  Output detrended data to this filename. Not implemented yet.
--version:
  Shows current version and build of this software
-v, --verbose:
  Print verbose messages. Specify multiple times to increase verbosity.
--references:
  Shows references to papers and sources.
--licenses:
  Shows licenses of code components used in this software.
```

## Example: Try it on a sample file: etp\_1kyr\_36Ma.txt

```
user$ ./hpnaff etp_1kyr_36Ma.txt -d 1 -m 75
```

```
=====
# | chunk |      t0 | points | frequency | amplitude | phase | significance |
period | arcsec/yr | phase (deg)
0 | 0 | 0.0000 | 14001 | 0.000000000 | 0.00000000 | 0.000000000 | 0.000000000 |
inf | 0.0000000 | 0.0000000
1 | 0 | 0.0000 | 14001 | 0.002472456 | 0.75721258 | 3.740563127 | 0.795224850 |
404.45610756 | 3.2043032 | 214.3184802
2 | 0 | 0.0000 | 14001 | 0.010522542 | 0.59639514 | -0.083030087 | 0.840326031 |
95.03406718 | 13.6372149 | -4.7572736
3 | 0 | 0.0000 | 14001 | 0.024999241 | 0.53877772 | 5.657761056 | 0.827374133 |
40.00121445 | 32.3990163 | 324.1658300
4 | 0 | 0.0000 | 14001 | 0.008040668 | 0.50803290 | 2.884638325 | 0.832167805 |
124.36778406 | 10.4207051 | 165.2776014
```

|               |            |             |       |             |            |              |             |
|---------------|------------|-------------|-------|-------------|------------|--------------|-------------|
| 5             | 0          | 0.0000      | 14001 | 0.010102115 | 0.39766702 | 4.191460276  | 0.868275179 |
| 98.98917382   | 13.0923408 | 240.1529838 |       |             |            |              |             |
| 6             | 0          | 0.0000      | 14001 | 0.042828239 | 0.31115152 | 1.466354530  | 0.901608023 |
| 23.34908056   | 55.5053976 | 84.0159259  |       |             |            |              |             |
| 7             | 0          | 0.0000      | 14001 | 0.007627856 | 0.30706911 | 0.558756463  | 0.900168886 |
| 131.09844523  | 9.8857008  | 32.0143871  |       |             |            |              |             |
| 8             | 0          | 0.0000      | 14001 | 0.000407279 | 0.27097231 | 2.625060814  | 0.917986628 |
| 2455.31930796 | 0.5278336  | 150.4049056 |       |             |            |              |             |
| 9             | 0          | 0.0000      | 14001 | 0.045302069 | 0.26148887 | 5.142663822  | 0.908826103 |
| 22.07404698   | 58.7114815 | 294.6529325 |       |             |            |              |             |
| 10            | 0          | 0.0000      | 14001 | 0.025832617 | 0.23907704 | 3.749692849  | 0.917331368 |
| 38.71075060   | 33.4790718 | 214.8415747 |       |             |            |              |             |
| 11            | 0          | 0.0000      | 14001 | 0.001034652 | 0.22887920 | 0.316263467  | 0.926607014 |
| 966.50814962  | 1.3409095  | 18.1205619  |       |             |            |              |             |
| 12            | 0          | 0.0000      | 14001 | 0.053350844 | 0.23311864 | 1.355155680  | 0.912974999 |
| 18.74384580   | 69.1426943 | 77.6447010  |       |             |            |              |             |
| 13            | 0          | 0.0000      | 14001 | 0.009911387 | 0.18079485 | 5.692179885  | 0.941223401 |
| 100.89404923  | 12.8451580 | 326.1378837 |       |             |            |              |             |
| 14            | 0          | 0.0000      | 14001 | 0.019210470 | 0.15673509 | -0.475810951 | 0.949106113 |
| 52.05494759   | 24.8967689 | -27.2619593 |       |             |            |              |             |
| 15            | 0          | 0.0000      | 14001 | 0.052929481 | 0.14841497 | 5.755575872  | 0.947991689 |
| 18.89306257   | 68.5966076 | 329.7702061 |       |             |            |              |             |
| 16            | 0          | 0.0000      | 14001 | 0.009488271 | 0.15112041 | -0.166596764 | 0.960411285 |
| 105.39327551  | 12.2967997 | -9.5452915  |       |             |            |              |             |
| 17            | 0          | 0.0000      | 14001 | 0.025098798 | 0.17311913 | 2.629896209  | 0.958954086 |
| 39.84254538   | 32.5280423 | 150.6819533 |       |             |            |              |             |
| 18            | 0          | 0.0000      | 14001 | 0.025414340 | 0.14355863 | 4.818018103  | 0.948646430 |
| 39.34786366   | 32.9369851 | 276.0521029 |       |             |            |              |             |
| 19            | 0          | 0.0000      | 14001 | 0.024888808 | 0.14098312 | 5.710170222  | 0.964800625 |
| 40.17870229   | 32.2558949 | 327.1686540 |       |             |            |              |             |
| 20            | 0          | 0.0000      | 14001 | 0.010323185 | 0.13699989 | -1.010239654 | 0.950068676 |
| 96.86932826   | 13.3788478 | -57.8824685 |       |             |            |              |             |
| 21            | 0          | 0.0000      | 14001 | 0.007440236 | 0.12986997 | 2.040413354  | 0.952028440 |
| 134.40433245  | 9.6425463  | 116.9070736 |       |             |            |              |             |
| 22            | 0          | 0.0000      | 14001 | 0.001455303 | 0.12708837 | 2.315320953  | 0.947284449 |
| 687.14223688  | 1.8860724  | 132.6581188 |       |             |            |              |             |
| 23            | 0          | 0.0000      | 14001 | 0.010925100 | 0.11817448 | -0.359553604 | 0.955088752 |
| 91.53234018   | 14.1589300 | -20.6009040 |       |             |            |              |             |
| 24            | 0          | 0.0000      | 14001 | 0.009667251 | 0.11874251 | 2.622499903  | 0.954076811 |
| 103.44202802  | 12.5287567 | 150.2581762 |       |             |            |              |             |
| 25            | 0          | 0.0000      | 14001 | 0.010413463 | 0.09715315 | 2.916722459  | 0.980490597 |
| 96.02953115   | 13.4958485 | 167.1158869 |       |             |            |              |             |
| 26            | 0          | 0.0000      | 14001 | 0.000901134 | 0.09569726 | 1.601650029  | 0.968540900 |
| 1109.71245232 | 1.1678701  | 91.7677869  |       |             |            |              |             |
| 27            | 0          | 0.0000      | 14001 | 0.012994167 | 0.10086003 | 0.611296497  | 0.967944111 |
| 76.95760564   | 16.8404408 | 35.0247093  |       |             |            |              |             |
| 28            | 0          | 0.0000      | 14001 | 0.042724400 | 0.09486676 | 1.049712893  | 0.980633322 |
| 23.40582906   | 55.3708222 | 60.1441185  |       |             |            |              |             |
| 29            | 0          | 0.0000      | 14001 | 0.000231340 | 0.09813349 | 0.120421080  | 0.967044518 |
| 4322.64694648 | 0.2998163  | 6.8996196   |       |             |            |              |             |
| 30            | 0          | 0.0000      | 14001 | 0.005563540 | 0.10009924 | 2.397732816  | 0.967425007 |
| 179.74168484  | 7.2103475  | 137.3799708 |       |             |            |              |             |
| 31            | 0          | 0.0000      | 14001 | 0.010638958 | 0.09688648 | -0.508419978 | 0.964555303 |
| 93.99417100   | 13.7880891 | -29.1303190 |       |             |            |              |             |
| 32            | 0          | 0.0000      | 14001 | 0.008918727 | 0.08741944 | 2.593049312  | 0.963917464 |
| 112.12362149  | 11.5586705 | 148.5707817 |       |             |            |              |             |
| 33            | 0          | 0.0000      | 14001 | 0.007853668 | 0.09431921 | 1.158980960  | 0.966015093 |
| 127.32903632  | 10.1783540 | 66.4047175  |       |             |            |              |             |
| 34            | 0          | 0.0000      | 14001 | 0.024592220 | 0.08828171 | 3.104291584  | 0.962948350 |
| 40.66326600   | 31.8715176 | 177.8628061 |       |             |            |              |             |
| 35            | 0          | 0.0000      | 14001 | 0.008385587 | 0.08641115 | 2.992612020  | 0.964466604 |
| 119.25223732  | 10.8677206 | 171.4640385 |       |             |            |              |             |
| 36            | 0          | 0.0000      | 14001 | 0.018145511 | 0.08763660 | 3.810401603  | 0.965100179 |
| 55.11004768   | 23.5165828 | 218.3199301 |       |             |            |              |             |
| 37            | 0          | 0.0000      | 14001 | 0.024810831 | 0.08240291 | 4.521072773  | 0.982190253 |
| 40.30497880   | 32.1548364 | 259.0383888 |       |             |            |              |             |
| 38            | 0          | 0.0000      | 14001 | 0.042929640 | 0.09470460 | -1.714770886 | 0.974168391 |
| 23.29392958   | 55.6368128 | -98.2491346 |       |             |            |              |             |
| 39            | 0          | 0.0000      | 14001 | 0.001312871 | 0.08074684 | 0.777678607  | 0.975863805 |
| 761.68923898  | 1.7014813  | 44.5577020  |       |             |            |              |             |
| 40            | 0          | 0.0000      | 14001 | 0.002884917 | 0.08065197 | 2.947032331  | 0.969528197 |
| 346.63039200  | 3.7388528  | 168.8525146 |       |             |            |              |             |
| 41            | 0          | 0.0000      | 14001 | 0.009282092 | 0.07995912 | -0.902914836 | 0.959472393 |
| 107.73432964  | 12.0295917 | -51.7332094 |       |             |            |              |             |
| 42            | 0          | 0.0000      | 14001 | 0.002172887 | 0.07898195 | 1.985388299  | 0.966843584 |
| 460.21714558  | 2.8160620  | 113.7543702 |       |             |            |              |             |
| 43            | 0          | 0.0000      | 14001 | 0.025182288 | 0.07351399 | 1.270869502  | 0.989395181 |
| 39.71045117   | 32.6362447 | 72.8154588  |       |             |            |              |             |
| 44            | 0          | 0.0000      | 14001 | 0.002006049 | 0.07122474 | 1.007509645  | 0.970121706 |
| 498.49230512  | 2.5998395  | 57.7260505  |       |             |            |              |             |
| 45            | 0          | 0.0000      | 14001 | 0.008233354 | 0.07018462 | 4.902075131  | 0.962605563 |
| 121.45718541  | 10.6704267 | 280.8682159 |       |             |            |              |             |
| 46            | 0          | 0.0000      | 14001 | 0.043838032 | 0.06715352 | 3.232504867  | 0.965810782 |
| 22.81124276   | 56.8140900 | 185.2088861 |       |             |            |              |             |

|               |  |   |  |            |  |             |  |             |  |            |  |              |  |             |  |
|---------------|--|---|--|------------|--|-------------|--|-------------|--|------------|--|--------------|--|-------------|--|
| 47            |  | 0 |  | 0.0000     |  | 14001       |  | 0.007224019 |  | 0.06531452 |  | 4.276908703  |  | 0.978785703 |  |
| 138.42710917  |  |   |  | 9.3623280  |  | 245.0488180 |  |             |  |            |  |              |  |             |  |
| 48            |  | 0 |  | 0.0000     |  | 14001       |  | 0.045402169 |  | 0.07436495 |  | 2.063108445  |  | 0.977076727 |  |
| 22.02537938   |  |   |  | 58.8412112 |  | 118.2074066 |  |             |  |            |  |              |  |             |  |
| 49            |  | 0 |  | 0.0000     |  | 14001       |  | 0.002344771 |  | 0.06402237 |  | 1.101819579  |  | 0.974716549 |  |
| 426.48086343  |  |   |  | 3.0388233  |  | 63.1296117  |  |             |  |            |  |              |  |             |  |
| 50            |  | 0 |  | 0.0000     |  | 14001       |  | 0.034012471 |  | 0.06632261 |  | 0.311286623  |  | 0.964879805 |  |
| 29.40098080   |  |   |  | 44.0801621 |  | 17.8354097  |  |             |  |            |  |              |  |             |  |
| 51            |  | 0 |  | 0.0000     |  | 14001       |  | 0.000615717 |  | 0.06955959 |  | 0.324652566  |  | 0.966238480 |  |
| 1624.12260860 |  |   |  | 0.7979693  |  | 18.6012218  |  |             |  |            |  |              |  |             |  |
| 52            |  | 0 |  | 0.0000     |  | 14001       |  | 0.024999132 |  | 0.06125458 |  | 3.871230190  |  | 0.987944595 |  |
| 40.00138890   |  |   |  | 32.3988750 |  | 221.8051514 |  |             |  |            |  |              |  |             |  |
| 53            |  | 0 |  | 0.0000     |  | 14001       |  | 0.012584554 |  | 0.06352103 |  | 4.354136361  |  | 0.970616108 |  |
| 79.46248859   |  |   |  | 16.3095823 |  | 249.4736369 |  |             |  |            |  |              |  |             |  |
| 54            |  | 0 |  | 0.0000     |  | 14001       |  | 0.007042302 |  | 0.05954369 |  | 3.178070923  |  | 0.967975779 |  |
| 141.99901671  |  |   |  | 9.1268238  |  | 182.0900509 |  |             |  |            |  |              |  |             |  |
| 55            |  | 0 |  | 0.0000     |  | 14001       |  | 0.053168977 |  | 0.06126971 |  | -0.361831203 |  | 0.974607869 |  |
| 18.80796025   |  |   |  | 68.9069938 |  | -20.7314008 |  |             |  |            |  |              |  |             |  |
| 56            |  | 0 |  | 0.0000     |  | 14001       |  | 0.018476857 |  | 0.05675901 |  | 1.358226583  |  | 0.977498040 |  |
| 54.12175935   |  |   |  | 23.9460065 |  | 77.8206508  |  |             |  |            |  |              |  |             |  |
| 57            |  | 0 |  | 0.0000     |  | 14001       |  | 0.025289765 |  | 0.06621217 |  | -0.908124254 |  | 0.973848255 |  |
| 39.54168736   |  |   |  | 32.7755361 |  | -52.0316871 |  |             |  |            |  |              |  |             |  |
| 58            |  | 0 |  | 0.0000     |  | 14001       |  | 0.045202487 |  | 0.05979910 |  | 4.539755712  |  | 0.986931113 |  |
| 22.12267664   |  |   |  | 58.5824230 |  | 260.1088423 |  |             |  |            |  |              |  |             |  |
| 59            |  | 0 |  | 0.0000     |  | 14001       |  | 0.001552070 |  | 0.05459046 |  | 3.384103161  |  | 0.981459103 |  |
| 644.30080647  |  |   |  | 2.0114828  |  | 193.8948286 |  |             |  |            |  |              |  |             |  |
| 60            |  | 0 |  | 0.0000     |  | 14001       |  | 0.010774958 |  | 0.06013873 |  | -0.920994981 |  | 0.970525666 |  |
| 92.80778711   |  |   |  | 13.9643455 |  | -52.7691253 |  |             |  |            |  |              |  |             |  |
| 61            |  | 0 |  | 0.0000     |  | 14001       |  | 0.035180014 |  | 0.05381168 |  | 6.108664303  |  | 0.971837152 |  |
| 28.42523015   |  |   |  | 45.5932984 |  | 350.0006830 |  |             |  |            |  |              |  |             |  |
| 62            |  | 0 |  | 0.0000     |  | 14001       |  | 0.025932854 |  | 0.05497186 |  | 0.528441367  |  | 0.981080756 |  |
| 38.56112370   |  |   |  | 33.6089791 |  | 30.2774600  |  |             |  |            |  |              |  |             |  |
| 63            |  | 0 |  | 0.0000     |  | 14001       |  | 0.019098347 |  | 0.05289607 |  | 5.666427635  |  | 0.977106362 |  |
| 52.36055271   |  |   |  | 24.7514576 |  | 324.6623884 |  |             |  |            |  |              |  |             |  |
| 64            |  | 0 |  | 0.0000     |  | 14001       |  | 0.052829237 |  | 0.05592637 |  | 4.622260192  |  | 0.978276065 |  |
| 18.92891225   |  |   |  | 68.4666917 |  | 264.8360008 |  |             |  |            |  |              |  |             |  |
| 65            |  | 0 |  | 0.0000     |  | 14001       |  | 0.011547573 |  | 0.05030718 |  | 3.661455364  |  | 0.969207741 |  |
| 86.59828253   |  |   |  | 14.9656548 |  | 209.7859392 |  |             |  |            |  |              |  |             |  |
| 66            |  | 0 |  | 0.0000     |  | 14001       |  | 0.052667555 |  | 0.04771644 |  | 4.565272429  |  | 0.975236135 |  |
| 18.98702155   |  |   |  | 68.2571512 |  | 261.5708425 |  |             |  |            |  |              |  |             |  |
| 67            |  | 0 |  | 0.0000     |  | 14001       |  | 0.008514638 |  | 0.04804064 |  | 4.938804770  |  | 0.978348985 |  |
| 117.44480713  |  |   |  | 11.0349707 |  | 282.9726692 |  |             |  |            |  |              |  |             |  |
| 68            |  | 0 |  | 0.0000     |  | 14001       |  | 0.017723966 |  | 0.04660213 |  | 1.785804511  |  | 0.976900743 |  |
| 56.42078067   |  |   |  | 22.9702600 |  | 102.3190615 |  |             |  |            |  |              |  |             |  |
| 69            |  | 0 |  | 0.0000     |  | 14001       |  | 0.013408861 |  | 0.04526629 |  | -0.218064195 |  | 0.976731996 |  |
| 74.57754915   |  |   |  | 17.3778840 |  | -12.4941580 |  |             |  |            |  |              |  |             |  |
| 70            |  | 0 |  | 0.0000     |  | 14001       |  | 0.004944881 |  | 0.04401069 |  | 3.792380702  |  | 0.977679719 |  |
| 202.22932980  |  |   |  | 6.4085660  |  | 217.2874085 |  |             |  |            |  |              |  |             |  |
| 71            |  | 0 |  | 0.0000     |  | 14001       |  | 0.024477668 |  | 0.04221972 |  | 2.795577253  |  | 0.982047896 |  |
| 40.85356574   |  |   |  | 31.7230571 |  | 160.1747779 |  |             |  |            |  |              |  |             |  |
| 72            |  | 0 |  | 0.0000     |  | 14001       |  | 0.025733760 |  | 0.04584982 |  | 3.159868039  |  | 0.992264403 |  |
| 38.85945885   |  |   |  | 33.3509534 |  | 181.0471025 |  |             |  |            |  |              |  |             |  |
| 73            |  | 0 |  | 0.0000     |  | 14001       |  | 0.044937430 |  | 0.04272239 |  | 0.214967789  |  | 0.979100871 |  |
| 22.25316390   |  |   |  | 58.2389096 |  | 12.3167471  |  |             |  |            |  |              |  |             |  |
| 74            |  | 0 |  | 0.0000     |  | 14001       |  | 0.018373301 |  | 0.04170326 |  | 4.267372537  |  | 0.976418419 |  |
| 54.42680232   |  |   |  | 23.8117976 |  | 244.5024360 |  |             |  |            |  |              |  |             |  |
| 75            |  | 0 |  | 0.0000     |  | 14001       |  | 0.016041231 |  | 0.04508949 |  | 2.028721000  |  | 0.980103195 |  |
| 62.33935561   |  |   |  | 20.7894353 |  | 116.2371511 |  |             |  |            |  |              |  |             |  |

## Notes

"Evolutive" outputs can be obtained with the -s option, e.g. -s 1000 computes an analysis for every 1000 points.

Source code available at repository below. Note: There are three branches, "main" contains a new swift package manager version, which can also be compiled for Linux/Windows, and "Accelerate" whichh uses the LinearAlgebra system library. The Linux version is linked statically. The "Windows" branch is modified to enable compilation with the Windows Swift Toolchain (see top).

<https://paloz.marum.de/bitbucket/scm/essp/hpnaff.git>

This is a Swift implementation of

<https://ops.aps.anl.gov/manuals/SDDStoolkit/SDDStoolkitsu61.html>

Related efforts:

<https://github.com/MichaelEhrlichman/FortNAFF>

<https://github.com/adrn/SuperFreq>

## References

1. Laskar, J., 1990, The chaotic motion of the Solar System. A numerical estimate of the size of the chaotic zones, *Icarus*, 88, 266-291.
2. Laskar, J., 1993, Frequency analysis for multi-dimensional systems. *Global dynamics and diffusion*, *Physica D*, 67, 257-281.
3. Dumas, S., Laskar, J., 1993, Global Dynamics and Long-Time Stability in Hamiltonian Systems Via Numerical Frequency Analysis, *Phys. Rev. Letters*, 70 (20), 2975-2979.
4. Laskar, J. : 1999, Introduction to frequency map analysis, in *proc. of NATO ASI 533 3DHAM95*, S'Agaro, Spain, 134150.
5. Papaphilippou, Y., Frequency maps for LHC models, PAC99.
6. Papaphilippou, Y. Zimmermann, F., Weak-strong beam-beam simulations for the Large Hadron Collider, *Phys. Rev. ST Accel. Beams* 2, 104001 (1999).
7. Robin, D., Steir, C., Laskar, J., Nadolski, L. : 2000, Global dynamics of the ALS revealed through experimental Frequency Map Analysis, *Phys. Rev. Lett.*, 85, pp. 558-561.
8. Laskar, J., Frequency map analysis and quasiperiodic decompositions. preprint (<https://arxiv.org/pdf/math/0305364.pdf>) (2003).
9. Valluri & Merritt, 1998